BASIC SLIDE RULE

DATA-GUIDE QUICK CHART

A SIMPLIFIED SUMMARY & INSTANT REFERENCE

LEARN & REVIEW IN MINUTES

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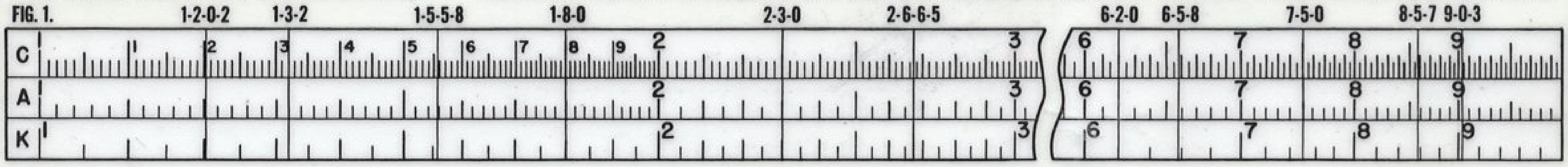
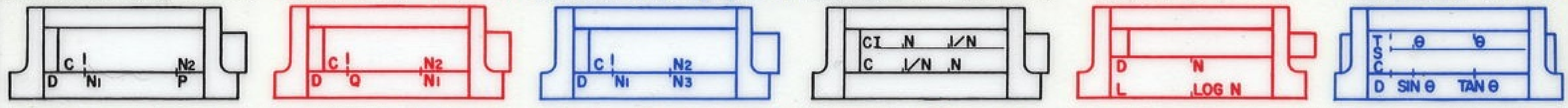
NOTE: The *C, A,* and *K* scales shown are not in their proper length proportion. All scales have been drawn to equal length to clearly illustrate the respective differences in scale division and to simplify the mastery of the number location method explained in Sec. **1**.

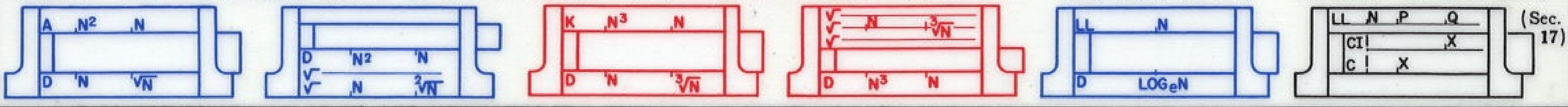
FIG. 1.



MULTIPLICATION: $N_1 \times N_2 = P$ (Sec. 3) — DIVISION: $N_1 \div N_2 = Q$ (Sec. 4) — PROPORTIONS: $l/N_1 = N_2/N_3$ (Sec. 6) — RECIPROCALS: $N \rightarrow 1/N$ (Sec. 8) — LOGS; BASE 10 (Sec. 15) — TRIG. FUNCTIONS (Sec. 16)



SQUARES: $N \rightarrow N^2$ (Sec. 10) AND SQUARE ROOTS: $N \rightarrow \sqrt{N}$ (Sec. 12) — CUBES: $N \rightarrow N^3$ (Sec. 11) AND CUBE ROOTS: $N \rightarrow \sqrt[3]{N}$ (Sec. 13) — LOGS; BASE e (Sec. 17) — POWERS: $N^x = P$; ROOTS: $\sqrt[x]{N} = Q$ (Sec. 17)



1 READING THE SLIDE RULE SCALES

A. THE DECIMAL POINT (d.p.). There is no way of indicating the position of the d.p. in a number read on a slide rule scale. Only the sequence of *significant digits* is indicated.

B. SIGNIFICANT DIGITS (s.d.'s.) in a number are the first non-zero digit and those following it, up to and including the last non-zero digit. **Ex:** s.d.'s in italics: *600, 450, 40500*, 0.0*600*, 0.*6020*, 0.00*160060*. Thus the numbers 10,500, 1,050, and .00105 are treated as the same number consisting only of the three s.d.'s 1-0-5. Likewise .002, .02, 2, 200, 2,000,etc. are treated as a number with the single s.d. 2. **NOTE:** Numbers may be expressed to three places using zeros when necessary since slide rule scales can usually be read accurately to just three places. **Ex:** 2 may be written 2-0-0; 69 as 6-9-0; etc. For the location of the d.p. in the answer see Section 2.

C-1. PRIMARY MARKS AND THE FIRST SIGNIFICANT DIGIT. The ten primary marks on each of the scales in Fig. 1 are labeled with the largest numbers (1, 2, 3, 4, 5, 6, 7, 8, 9, 1) and divide the *length of a scale into nine primary spaces*. The scale may run the length of the rule (*C* scale) or may be repeated several times (*A* and *K* scales) as seen on your slide rule.

C-2. A NUMBER WITH ONE SIGNIFICANT DIGIT IS LOCATED AT THE CORRESPONDING PRIMARY MARK. **Ex:** No.'s 1-0-0, 2-0-0, 3-0-0, etc. are located at primary marks 1, 2, 3, etc.

D-1. SECONDARY MARKS AND THE SECOND SIGNIFICANT DIGIT. Secondary marks (the heavy lines in Fig. 1) form the major divisions of the space between two consecutive primary marks and may vary in number. First in Fig. 1 and on your slide rule that nine secondary marks form ten secondary spaces. Each space is then equal to one unit in the second place of a number and s.d.'s 1 to 9 can be located on secondary marks. Four marks form only five secondary spaces, each space representing two units in the second place. This means that even digits (2, 4, 6, 8) are located on marks and odd digits (1, 3, 5, 7, 9) are located midway between marks.

D-2. A NUMBER WITH TWO SIGNIFICANT DIGITS IS LOCATED AT THE SECONDARY MARK OR IN THE SECONDARY SPACE REPRESENTING THE SECOND DIGIT FOLLOWING THE PRIMARY MARK THAT REPRESENTS THE FIRST DIGIT.

Ex: 1. No. 1-8-0 is located on each scale at the eighth secondary mark following primary mark 1.

Ex: 2. No. 2-3-0 is located on each scale at the third secondary mark after primary mark 2.

Ex: 3. No. 6-2-0 on scale *K* is located at the first secondary mark after primary mark 6.

Ex: 4. 7-5-0 on scale *K* is located halfway between second secondary mark (7-4-0) and third secondary mark (7-6-0).

E-1. TERTIARY MARKS AND THE THIRD DIGIT. Tertiary marks (the thin lines in Fig. 1) divide the space between two consecutive secondary marks.

E-2. A NUMBER WITH THREE SIGNIFICANT DIGITS IS LOCATED AT THE CORRESPONDING TERTIARY MARK OR IN THE TERTIARY SPACE FOLLOWING THE SECOND DIGIT POSITION.

Ex: 5. No. 1-3-2. On scale *C* it is located at the second tertiary mark following position 1-3-0. On scale *A* it is located at the first tertiary mark following position 1-3-0, since each space is valued two units. On scale *K*, 1-3-2 is estimated two-fifths of the way between 1-3-0 and the next tertiary mark 1-3-5.

Ex: 6. No. 8-5-7. On scale *C* it is located two-fifths of the way between the center tertiary mark 8-5-5 and mark 8-6-0. On scale *A*, 8-5-7 is estimated at a point seven-tenths of the way between 8-5-0 and 8-6-0 since the entire space is equivalent to 10 units in the third place. On scale *K* first locate 8-5-0 midway between marks 8-4-0 and 8-6-0. Then estimate a point seven-tenths of the way between 8-5-0 and 8-6-0 to locate 8-5-7.

Ex: 7. No. 9-0-3. **NOTE:** When the second digit is zero the number is always located in the space between the appropriate primary mark and first secondary mark following it. No. 9-0-3. On scale *C*, located three-fifths of the way between primary mark 9 (9-0-0) and the first tertiary mark (9-0-5). On scale *A*, since there are no tertiary marks, it is located three-tenths of the way between primary mark 9 (9-0-0) and the first secondary mark (9-1-0). On scale *K*, first estimate 9-1-0; then estimate 9-0-3 at a point three-tenths of the distance between 9-0-0 and 9-1-0.

F. THE FOURTH DIGIT OF A NUMBER can be located only on that portion of a scale containing ten tertiary spaces.

Ex: 8. No. 1-5-5-8 is located on scale *C* eight-tenths of the way between 1-5-5-0 and 1-5-6-0.

Ex: 9. No. 1-2-0-2. **NOTE:** When the third digit is zero, the number is always located in the space between the appropriate secondary mark and the first tertiary mark following it. No. 1-2-0-2 on scale *C* is two-tenths of the way between 1-2-0-0 and 1-2-1-0.

G. IF A NUMBER HAS MORE SIGNIFICANT DIGITS THAN CAN BE LOCATED ACCURATELY ON A GIVEN SCALE, it is first rounded off. (See Sec. 2.)

SUMMARY OF VARIATIONS IN SLIDE RULE SCALE DIVISIONS.

MARKS	SPACES	SPACE/VALUE	DIGIT LOCATION
9	10	1 unit	all digits located on marks
4	5	2 units	2, 4, 6, 8 on marks; 1, 3, 5, 7, 9 half-way between marks.
1	2	5 units	5 on the mark; 1 to 4 and 6 to 9 estimated in space left or right of the mark.
0	1	10 units	all digits estimated by approximate division of the space into ten parts.

2 LOCATION OF DECIMAL POINT

Found by obtaining an approximate answer (**AA**) as follows:

STEP 1. ROUND OFF. Set all s.d.'s but the first equal to "0". **Ex:** 1340 becomes 1000; .0609⟶.06; 53.65⟶50; .003006⟶.003. **NOTE:** Increase the first s.d. by one unit if the second is five or more. **Ex:** 4.62⟶5; .987⟶1.0

STEP 2. CONVERT TO POWERS OF TEN.

A. POWERS OF TEN $10^0 = 1$; $10^1 = 10$; $10^2 = 100$; $10^3 = 1000$; etc. $10^{-1} = 0.1$; $10^{-2} = 0.01$; $10^{-3} = 0.001$; etc.

NOTE: Negative powers of ten are the reciprocals of the corresponding powers of ten. **Ex:** $10^{-2} = 1/10^2 = 1/100 = .01$

B. CONVERSION FORM. Numbers are written with one (or two) s.d.'s to the left of the d.p. **Ex:** $300 = 3 \times 10 \times 10 = 3 \times 10^2$ The integer is called the multiplier. The exponent of ten is the power. **1. Numbers > 10:** Shift the d.p. "m" places left, multiply by 10^m. **Ex:** $500 = 5 \times 10^2$; $6000 = 6 \times 10^3$ **2. Numbers < 1:** Shift the d.p. "m" places right and multiply by 10^{-m}. **Ex:** $0.06 = 6 \times 10^{-2}$; $0.0030 = 3 \times 10^{-3}$

STEP 3. PERFORM INDICATED OPERATIONS.

A. MULTIPLICATION. $(N_1 \times 10^x)(N_2 \times 10^y) = (N_1 \times N_2)(10^{x+y})$. Exponents are added algebraically. **Ex:** $(6 \times 10^2)(3 \times 10^3) = 18 \times 10^5$ **Ex:** $(4 \times 10^3)(3 \times 10^{-2}) = 12 \times 10^1$ **Ex:** $(7 \times 10^1)(2 \times 10^{-4}) = 14 \times 10^{-3}$

B. DIVISION. $(N_1 \times 10^x) \div (N_2 \times 10^y) = (N_1 \div N_2)(10^{x-y})$. Exponents are subtracted algebraically. **Ex:** $(6 \times 10^3) \div (3 \times 10^2) = 2 \times 10^1$ **Ex:** $(8 \times 10^4) \div (4 \times 10^4) = 2 \times 10^0 = 2$ **Ex:** $(9 \times 10^1) \div (3 \times 10^{-5}) = 3 \times 10^{(1)-(-5)} = 3 \times 10^6$

C. POWERS. $(N \times 10^x)^y = (N^y)(10^{xy})$. Exponents are multiplied algebraically. **Ex:** $(2 \times 10^3)^2 = 2^2 \times 10^{3 \times 2} = 4 \times 10^6$ **Ex:** $(2 \times 10^{-3})^3 = 8 \times 10^{-9}$

D. ROOTS. $\sqrt[y]{N \times 10^x} = \sqrt[y]{N} \times 10^{x/y}$. Exponents are divided algebraically. **Ex:** $\sqrt{400} = \sqrt{4 \times 10^2} = \sqrt{4} \times 10^{2/2} = 2 \times 10^1$ **Ex:** $\sqrt{.0004} = \sqrt{4 \times 10^{-4}} = 2 \times 10^{-2}$ **Ex:** $\sqrt[3]{27{,}000} = \sqrt[3]{27 \times 10^3} = 3 \times 10^1$

STEP 4. CONVERT ANSWERS BACK TO DECIMAL NOTATION.

A. $N \times 10^m$: Shift the d.p. "m" places right. **Ex:** $6 \times 10^3 = 6000$ **Ex:** $5.4 \times 10^1 = 54$ **B. $N \times 10^{-m}$:** Shift decimal point "m" places left. **Ex:** $7.14 \times 10^{-2} = 0.0714$ **Ex:** $.0600 \times 10^{-1} = .006$

C. COMBINED OPERATIONS.

$$\frac{3.02 \times 120 \times \sqrt{392}}{1.15 \times (30.6)^2} \quad AA: \frac{(3 \times 10^0)(1 \times 10^2)(\sqrt{4 \times 10^2})}{(1 \times 10^0)(3 \times 10^1)^2} = \frac{3 \times 1 \times 2}{1 \times 9} \times 10^{\left(0+2+\frac{2}{2}-0-(1 \times 2)\right)} = 2/3 \times 10^1 = 6.67$$

3 MULTIPLICATION OPERATIONS

A. SCALES USED. The *C* on the slide and the *D* on the body extend the length of the rule. The left and right "1" marks are called the left and right *C* and *D* indices. (*LC1, RC1, LD1, RD1*)

B. SLIDE RULE OPERATION (SRO): $N_1 \times N_2 = P$. **1.** Set the left *C* index over the first number N_1 on the *D* scale. **2.** Move the hairline (**hln**) to the second number N_2 on the *C* scale. **3.** Read the product P under the hairline on the *D* scale. **Ex:** 2×3 **SRO:** Set *LC1* over *D2*. Move hln to *C3*. Read product under the hln at *D6*. ANS: 6. **NOTE:** If in the second step, N_2 cannot be positioned on the *C* scale, set the right *C* index over N_1 instead. **Ex:** 2×9. **SRO:** If *LC1* is set over *D2*, *C9* is off scale; therefore, set *RC1* over *D2*. Move hln to *C9*. Read product under hln at *D1-8*. ANS: 18.

C. COMPLETE OPERATION. **Ex:** 21×320 **AA:** Round off and express as powers of ten; $21 \rightarrow 20 = 2 \times 10^1$; $320 \rightarrow 300 = 3 \times 10^2$. Perform the indicated operation; *multiply:* $(2 \times 10^1)(3 \times 10^2) = 6 \times 10^3 = 6000$. **SRO:** Set *LC1* on *D2-1*. Move hln to *C3-2*. Read the product under the hln at *D6-7-2*. ANS: Since *AA* 6000, the true answer is 6,720. **Ex:** 0.0855×4120 **AA:** $(9 \times 10^{-2})(4 \times 10^3) = 36 \times 10^1 = 360$ **SRO:** Set *RC1* on *D8-5-5*, hln to *C4-1-2*. Product under hln at *D3-5-2*. ANS: 352 **Ex:** 0.442×11.6 **AA:** $(4 \times 10^{-1})(1 \times 10^1) = 4 \times 10^0 = 4$ **SRO:** *LC1* on *D4-4-2*, hln to *C1-1-6*. Product under hln at *D5-1-2*. ANS: 5.12

D. PERCENTAGES. $x\%$ of $N = (x) \times (N) \times (10^{-2})$. **Ex:** 50% of $12 = 50 \times 12 \times 10^{-2} = 600 \times 10^{-2} = 6$ **Ex:** 6.4% of .036 **AA:** $6 \times (4 \times 10^{-2}) \times 10^{-2} = 24 \times 10^{-4} = .0024$ **SRO:** Set *RC1* over *D6-4*. Move hln to *C3-6*. Read under hln, *D2-3*. ANS: 0.0023

E. CONTINUED PRODUCTS. $N_1 \times N_2 \times N_3 = P$. **SRO: 1.** Multiply the first two factors $(N_1 \times N_2 = P_a)$. **2.** Multiply this partial product by the next factor $(P_a \times N_3 = P_b)$. **3.** Continue multiplying each partial product by the next factor until the final product is obtained. **Ex:** $6.4 \times .088 \times 12.3$ **AA:** $6 \times (9 \times 10^{-2}) \times (1 \times 10^1) = 5.4$ **SRO:** It is not necessary to read any of the partial products. Set *RC1* over *D6-4*. Move hln to *C8-8*. Bring *LC1* under the hln. Move hln to *C1-2-3*. The final product is under hln at *D6-9*. ANS: 6.9

4 DIVISION OPERATIONS

A. SCALES USED are *C* and *D*.

B. SRO: $N_1 \div N_2 = Q$. **1.** Set hairline over the numerator N_1 on *D* scale. **2.** Bring the denominator N_2 on the *C* scale under the hairline. **3.** Read quotient Q under whichever *C* index falls on the *D* scale. **Ex:** $6 \div 3$ **SRO:** Set hln on *D6*. Bring *C3* under hln. Read quotient under *LC1* at *D2*. ANS: 2 **Ex:** $62 \div 305$ **AA:** $(6 \times 10^1) \div (3 \times 10^2) = 2 \times 10^{-1} = 0.2$ **SRO:** Set hln on *D6-2*. Bring *C3-0-5* under hln. Read quotient under *LC1* at *D2-0-3*. ANS: 0.203 **Ex:** $17.55 \div 0.00203$ **AA:** $(2 \times 10^1) \div (2 \times 10^{-3}) = 1 \times 10^4 = 10{,}000$ **SRO:** hln on *D1-7-5-5*. Bring *C2-0-3* under hln. Under *RC1* read *D8-6-4*. ANS: 8,640

5 COMBINED MULTIPLICATION AND DIVISION

A. ONE FACTOR IN NUM. OR DENOM. Perform sequence mult. then division: $N_1 \times N_2 \times N_3 ..... \div N_d$ or $(N_n \div N_1) \times N_2 \times N_3 ....$

B. SEVERAL FACTORS IN NUM. AND DENOM. 1. Perform mult. and div. operations alternately whenever possible since the least number of slide rule motions will then be required. **2. Form:** $\frac{N_1 \times N_2}{N_3 \times N_4}$ perform as $N_1 \div N_3 \times N_2 \div N_4$; $\frac{N_1 \times N_2 \times N_3 \times N_4}{N_5 \times N_6}$ perform as $N_1 \times N_2 \div N_5 \times N_3 \div N_6 \times N_4$; $\frac{N_1 \times N_2}{N_3 \times N_4 \times N_5}$ perform as $N_1 \div N_3 \times N_2 \div N_4 \div N_5$; etc.

Ex: $\frac{2.02 \times 120 \times 0.0925}{1.15 \times 0.81}$ **AA:** $\frac{2 \times (1 \times 10^2) \times (9 \times 10^{-2})}{1 \times (8 \times 10^{-1})} =$ $18 \div 8 \times 10^1 \cong 20$ **SRO:** It is not necessary to read any intermediate results. Perform as $2\text{-}0\text{-}2 \div 1\text{-}1\text{-}5 \times 1\text{-}2\text{-}0 \div 8\text{-}1 \times 9\text{-}2\text{-}5$. *Divide:* Set hln on *D2-0-2*. Bring *C1-1-5* under hln. *Multiply:* Move hln to *C1-2-0*. *Divide:* Bring *C8-1-0* under hln. *Multiply:* Move hln to *C9-2-5*. Read *D2-4-1*. ANS: 24.1

6 PROPORTION OPERATIONS

A. PRINCIPLE. Any pair of numbers set opposite each other on the *C* and *D* (or *CF* and *DF*) scales are in the same proportion as any other pair of numbers found opposite each other along the entire length of the scales. **Ex:** Set *C1* over *D2*; the proportion is 1 to 2 (written also 1/2 or 1:2); read on the scales *C2* opposite *D4*, *C3* opposite *D6*, etc.

B. FORMATION OF PROPORTIONS. 1. A whole number may be divided by 1. **Ex:** $6 = 2.6/x$ solve as $6/1 = 2.6/x$. **2.** A factor in num. (*denom.*) of one ratio may be transferred to denom. (*num.*) of other. **Ex:** $6 = \frac{2.4 \times 9.4}{x}$ solve as $\frac{6}{9.4} = \frac{2.4}{x}$ or $\frac{x}{2.4} = \frac{9.4}{6}$

C. MULTIPLE PROPORTIONS. Ex: $\frac{.0202}{.182} = \frac{x}{.3} = \frac{4.5}{y}$ (Set nums on C) (Set denoms on D) **AA:** $x = (2 \times 10^{-2}) \times (3 \times 10^{-1}) \div (2 \times 10^{-1}) = 3 \times 10^{-2} = .03$; $y = (5) \times (2 \times 10^{-1}) \div (2 \times 10^{-2}) = 5 \times 10^1 = 50$. **SRO:** Set *C2-0-2* over *D1-8-2*. Move hln to *D3*; read *C3-3-3*. Move hln to *C4-5*; read *D4-0-5*. ANS: x = .0333; y = 40.5

7 USING FOLDED SCALES

A. SCALES. The *CF*, a folded *C* scale is on the slide; the *DF*, a folded *D* scale, is on the body. Both scales have a single index in the center. Their extreme right and left ends are labeled π.

B. PRINCIPLE. Operations performed on the *C* and *D* scales are simultaneously being performed on the *CF* and *DF* scales.

C. APPLICATIONS. 1. When numbers can not be conveniently matched because of position on C - D scales, the operation can be transferred to the folded scales. The calculation can either be completed on the folded scales or returned to the *C* and *D* scales at will. **NOTE:** When the answer is to be found under an index, it can be read under either the *CF* or *C* indices; but when the answer is to be found under the hairline, it must be read on the scale concerned. **Ex:** $(18 \div 4) \times 11$. **AA:** $(20 \div 4) \times 10 = 50$. **SRO:** *Divide:* Over *D1-8* set *C4*; quotient is then under *RC1* at *D4-5*. *Multiply:* Move hln to *C1-1*; since 1-1 is off scale, it would be necessary to switch LC1 over 4-5 to bring 1-1 back on scale. Instead, note DF4-5 is over CF1. Then, merely move hln to CF1-1 and read under hln, DF4-9-5. ANS: 49.5

2. Multiplication by π. SRO: Set N on D under hln, read $\pi \times$ N on DF. **Ex:** Opposite 3 on D read $3\pi = 9.42$ on DF.

3. Division by π. SRO: Set N on DF under hln, read N $\div \pi$ on D. Opposite 4 on DF read $4/\pi = 1.273$ on D. **NOTE:** Circumference (on DF) $= \pi \times$ diameter (on D); diameter (on D) = circumference (on DF)/π.

8 RECIPROCAL (or INVERSE) SCALES

A. SCALES. The CI, on the slide, is an inverted C scale, and reads from right to left. The CIF (above the CI) is a folded CI scale having a single index in the center of the scale.

B. RECIPROCALS. 1. Definition. The reciprocal of the number N equals 1/N; also written as N^{-1}. **Ex:** $5^{-1} = 1/5 = .200$ **2. Properties.** Division by N can be replaced by multiplication by 1/N. a $\div$ N = a $\times$ 1/N. **Ex:** $6 \div 3 = 6 \times 1/3 = 2$ Multiplication by N can be replaced by division by 1/N. a $\times$ N = a $\div$ 1/N. **Ex:** $4 \times 2 = 4 \div 1/2 = 8$

C. USING RECIPROCAL SCALES. 1. To obtain a reciprocal. SRO: Set N on scale C under the hairline, read 1/N on the CI scale If N is on the CF, 1/N is read from CIF scale. **Ex:** Find 1/246 AA: $1/246 \rightarrow 1/200 = 1/2 \times 10^{-2} = .5 \times 10^{-2} = .005$ SRO: Set C2-4-6 under hln and read CI4-0-6 ANS: 0.00406

2. To reduce slide motions in mult. and div. when factors are widely separated. *Remember:* When using CI, answers are read on D; with the CIF, answers are read on the DF. **Ex:** $12 \div 7.5$ Perform as 12×7.5^{-1} SRO: Set LC1 over D1-2. Move hln to CI7-5. Read under hln, D1-6. ANS: 1.6 **Ex:** 12×9.1 Perform as $12 \div 9.1^{-1}$ SRO: Set hln over D1-2. Bring CI9-1 under hln. Read under RC1, D1-0-9-2. ANS: 109.2 **NOTE:** 12×9.1 could also have been calculated with the CF and DF.

3. To simplify combined operations by permitting complete alternation of mult. and div. operations. NOTE: The proficient use of the CI and CIF scales is of fundamental importance in fast computation. **Ex:** $2.1 \times 14 \times 6.6 \div 0.073$ AA: $(2 \times 10 \times 7) \div (7 \times 10^{-2}) = 140 \div 7 \times 10^{-2} = 20 \times 10^2 = 2000$ SRO: Perform as $2.1 \div 1.4^{-1} \div 6.6^{-1} \times 7.3^{-1}$ *Multiply:* Set LC1 over D2-1, move hln to C1-4. *Divide:* Bring CI6-6 under hln. *Multiply:* Set hln on CI7-3. Read under hln, D2-6-6. ANS: 2660

9 COMBINED OPERATIONS: FOLDED, INVERSE SCALES

NOTE: With the combined use of folded and inverse scales, a minimum number of slide and hln shifts is required. **Ex:** $2 \times 1.4 \times 6.2 \times 7 \times 0.51$ AA: $2 \times 1 \times 6 \times 7 \times (5 \times 10^{-1}) = 42$ SRO. Perform as $2 \times 1.4 \div 6.2^{-1} \times 7 \div 0.51^{-1}$ *Multiply:* Set LC1 on D2, hln on C1-4. *Divide:* Bring CI6-2 under hln. *Multiply:* Since C7 is off scale, set hln on CF7. *Divide:* Bring CIF5-1 under hln. Read under CF index, DF6-2. ANS: 62 $6.04 \times .051 \times 86 \div 2.64$ AA: $6 \times (5 \times 10^{-2}) \times 8 \times 10 \div 2 = 12$ SRO: Perform as $6.04 \times .051 \div 86^{-1} \times 2.64^{-1}$ *Multiply:* Set LC1 at D6-0-4, hln on C5-1. *Divide:* Bring CI8-6 under hln. *Multiply:* Set hln on CIF2-6-4. Read under hln, DF1-0-0-2. ANS: 10.02

10 HOW TO FIND SQUARES

A. SCALES USED. D and C with the A and B [or $\sqrt{\ }$] scales. The A on the body and B on the slide each contain two scales, similar to, but ½ the length of the D. [Two full-length $\sqrt{\ }$ scales may replace the A and B.]

B. SRO: $N \rightarrow N^2$. Set hairline over N on D scale and read N^2 under hairline on A scale (or set hairline over N on C scale and read N^2 under hairline on B scale). **Ex:** 3^2 SRO: Set hln over D3, read under hln, A9. ANS: 9 **Ex:** $(41)^2$ AA: $(4 \times 10^1)^2 = 16 \times 10^2 = 1600$ SRO: hln on D4-1, read under hln, A1-6-8. ANS: 1680. **Ex:** $(.115)^2$ AA: $(1 \times 10^{-1})^2 = 1 \times 10^{-2} = .01$ SRO: hln on D1-1-5; read under hln, A1-3-2. ANS: 0.0132 **Ex:** $(613)^2$ AA: $(6 \times 10^2)^2 = 36 \times 10^4 = 360{,}000$ SRO: hln on D6-1-3; read under hln, A3-7-6. ANS: 376,000.

C. SRO: 1. Set hairline on N on $\sqrt{\ }$ scale. **2.** Read N^2 on D. **Ex:** 6.1^2 SRO: Set hln on 6-1 on $\sqrt{\ }$ scale; read D3-7-2. ANS: 37.2

11 HOW TO FIND CUBES

A. SCALES USED. The D with K [or $\sqrt[3]{\ }$] scales. The K on the body is composed of three scales, similar to, but each ⅓ the length of the D. [Three full-length $\sqrt[3]{\ }$ scales may replace the K.]

B. SRO: $N \rightarrow N^3$. Set hairline over N on D scale; read N^3 under hairline on K scale. **Ex:** 2^3 SRO: Set hln over D2; read under hln, K8. ANS: 8 **Ex:** $(0.117)^3$ AA: $(0.117)^3 = (1 \times 10^{-1})^3 = .001$ SRO: hln on D1-1-7, read under hln, K1-6-1 ANS: .00161

12 SQUARE ROOT OPERATIONS

A. SCALES USED. Same as for squares.

B. FORM OF N. 1. Must have either one or two digits to the left of the d.p. **2.** Otherwise, re-write N with an even power of ten ($10^{\cdots -4, -2, 0, 2, 4 \cdots}$) such that one or two digits are placed left of the d.p. in the multiplier. Then, take the square root of this power of ten. **Ex:** $\sqrt{400} = \sqrt{4 \times 10^2} = \sqrt{4} \times 10$. **Ex:** $\sqrt{0.00225} = \sqrt{22.5 \times 10^{-4}} = \sqrt{22.5} \times 10^{-2}$. The square root of the multiplier is found on the slide rule as follows.

C. SRO: $N \rightarrow \sqrt{N}$. **1.** Set the hairline over N located on the proper section of the A or B scales as shown:

No. of digits left of d.p. in multiplier	1	2
Root location on A or B scale	left (1st)	right (2nd)

2. Read $\sqrt{N}$ under the hairline on the D scale if N is set on A (or on the C scale is N is set on B).

D. SRO: 1. Set hairline on N on D. **2.** Read $\sqrt{N}$ on upper or lower $\sqrt{\ }$ scale if N has one or two digits left of the d.p.

E. DECIMAL POINT LOCATION. 1. Always place the d.p. after the first digit of the number read from the slide rule. **2.** When the square root is multiplied by a power of ten, move the d.p. the indicated number of places. **Ex:** $\sqrt{900}$ *Form:* $\sqrt{900} = \sqrt{9} \times 10^2 = \sqrt{9} \times 10^1$ SRO: hln on A9 (left section). Read under hln D3-0-0. D.P.: 3.0×10^1 ANS: 30 **Ex:** $\sqrt{25}$ *Form:* $\sqrt{25}$ is already in the proper form. SRO: Set hln on A2-5 (right section). Read under hln, D5-0-0. D.P.: 5.0 ANS: 5.0 **Ex:** $\sqrt{415} = \sqrt{4.15 \times 10^2} = \sqrt{4.15} \times 10^1$ SRO: Set hln on A4-1-5 (left section). Read under hln, D2-0-4. D.P.: 2.04×10^1 ANS: 20.4 **Ex:** $\sqrt{0.00365} = \sqrt{36.5 \times 10^{-4}} = \sqrt{36.5} \times 10^{-2}$ SRO: Set hln on A3-6-5 (right). Read D6-0-4. D.P.: 6.04×10^{-2} ANS: 0.0604 **Ex:** $\sqrt{52.4}$ SRO: Set hln on D5-2-4; read 7-2-4 on lower $\sqrt{\ }$ scale. ANS: 7.24

13 CUBE ROOT OPERATIONS

A. SCALES USED. Same as for cubes.

B. FORM OF N. 1. Must have either one, two, or three digits to the left of the d.p. **2.** Otherwise, rewrite N with a power of ten that is a multiple of three ($10^{\cdots -6, -3, 0, 3, 6 \cdots}$) such that one, two or three digits are placed to the left of the d.p. in the multiplier. Then, take the cube root of this power of ten. **Ex:** $\sqrt[3]{27000} = \sqrt[3]{27 \times 10^3} = \sqrt[3]{27} \times 10^1$; $\sqrt[3]{.0000157} = \sqrt[3]{15.7 \times 10^{-6}} = \sqrt[3]{15.7} \times 10^{-2}$. The cube root of the multiplier is found on the slide rule as follows.

C. SRO: $N \rightarrow \sqrt[3]{N}$ Set the hairline over N on the proper section of the K scale as shown:

Digits left of the d.p. in the multiplier	1	2	3
Root location on section of K scale	left (1st)	middle (2nd)	right (3rd)

2. Read $\sqrt[3]{N}$ under the hairline on the D scale.

D. SRO: 1. Set N on D. Read $\sqrt[3]{N}$ on upper, middle or lower $\sqrt[3]{\ }$ scale if N has one, two, or three digits left of the d.p.

E. THE LOCATION OF THE DECIMAL POINT. 1. Always place the d.p. after the first digit of the number read from the slide rule. **2.** When the cube root is multiplied by a power of ten, move the d.p. the indicated number of places. **Ex:** $\sqrt[3]{4150}$ *Form:* $\sqrt[3]{4150} = \sqrt[3]{4.15 \times 10^3} = \sqrt[3]{4.15} \times 10^1$. SRO: Set hairline on K4-1-5 (left section). Read under hln, D1-6-0-5. D.P.: 1.605×10^1 ANS: 16.05 **Ex:** $\sqrt[3]{0.000068}$ *Form:* $\sqrt[3]{0.000068} = \sqrt[3]{68.0 \times 10^{-6}} = \sqrt[3]{68.0} \times 10^{-2}$. SRO: Set hln on K6-8-0 (middle section). Read D4-0-8. D.P.: $4.08 \times 10^{-2} = 0.0408$. [**Ex:** $\sqrt[3]{47.3}$ SRO: Set hln on D4-7-3; read 3-6-1-5 on middle $\sqrt[3]{\ }$ scale. ANS: 3.615]

14 COMBINED OPERATIONS: SQUARES, SQUARE ROOTS

These problems can be solved without first finding the required roots or powers using the following methods.

A. SQUARE ROOTS. 1. Write all roots in the proper form (see **12**:B). **2.** Perform the mult. and div. operations in the usual manner on the scales previously described but *when the square root of a number N is needed:* ■ on the C (moving) scale, set the hairline over N on the proper section of the B (moving) scale; ■ on the D (fixed) scale, set the hairline over N on the proper section of the A (fixed) scale. **Ex:** $2 \times \sqrt{9}$ SRO: Set LC1 at D2. Since the root is required on the C scale set hln on B9 (left section). Read under hln, D6. ANS: 6 **Ex:** $\sqrt{16} \div 2 = 2$ SRO: Since root is required on the D scale set hln on A1-6 (right section). Bring C2 under hln. Read under LC1, D2. ANS: 2 **Ex:** $2.06 \times \sqrt{.062} \div \sqrt{916}$. *Form:* $2.06 \times \sqrt{6.2 \times 10^{-1}} \div \sqrt{9.16 \times 10^1}$. AA: $2 \times (2 \times 10^{-1}) \div (3 \times 10^1) \cong .01$ SRO: Set LC1 at D2-0-6. Set hln over B6-2 (left section). Bring B9-1-6 (left section) under hln. Read under LC1, D1-6-9-5. ANS: 0.01695

B. SQUARES. 1. Mult. and div. may be performed on the A and B scales exactly as on the C and D scales. **2.** *When the square of a number N is needed:* ■ on the B (moving) scale, set the hairline over N on the C (moving) scale; ■ on the A (fixed) scale, set the hairline over N on the D (fixed) scale. **Ex:** 2×3^2 SRO: Set LB1 at A2. Set hln on C3. Read under hln, A1-8. ANS: 18 **Ex:** $6^2 \div 4$ SRO: Set hln on D6. Bring B4 under hln. Read under B1, A9. ANS: 9 **Ex:** $(4.1)^2 \div (6.8)^2 \times 2$ AA: $20 \div 50 \times 2 = 0.800$ SRO: Set hln on D4-1. Bring C6-8 under hln. Set hln on B2 and read under hln A7-2-7. ANS: 0.727

C. MIXED SQUARE ROOTS AND SQUARES. For the roots, use the rules shown in A, but compute squares as a repeated product (see Sec. 3:E). **Ex:** $6 \times \sqrt{9} \times 2^2$ performed as $6 \times \sqrt{9} \times 2 \times 2$.

15 LOGARITHM OPERATIONS

A. SCALES USED. The L and D both on the body. The L scale has eleven primary marks labeled, 0, 0.1, 0.2, 0.3, etc. to 1 forming 10 equal primary spaces. The value of secondary and tertiary divisions is similar to the other scales.

B. COMPUTING LOGS. 1. To determine the characteristic, C: Write N in powers of ten placing one digit to the left of the d.p. of the multiplier. C = the value of the exponent. **2. To determine the mantissa, M:** SRO: Set the hairline over N on D scale, read M under the hairline on L scale. **Ex:** log 36; C: $36 = 3.6 \times 10^1$; C = +1. M: Set hairline on D3-6; read M = 0.556 on L. ANS: log 36 = 1 + .556 = 1.556 **Ex:** log 0.0622; C: $.0622 = 6.22 \times 10^{-2}$; C = −2. M: Set hairline on D6-2-2; read M = 0.794 on L. ANS: log 0.0622 = −2 + 0.794 written as 8.794 − 10 or $\bar{2}$.794.

C. ANTILOGS. Given the logarithm of N, find N. SRO: **1.** Set the mantissa on L, read N on D. **2.** D.P. location: Place the d.p. to the right of the first digit read from the slide rule. Convert the characteristic to a power of ten then move the d.p. the indicated number of places. **Ex:** Given log N = 9.583 − 10 find N. SRO: Set .583 on L, read under hln D3-8-3. D.P.: Write 3.83. Since C = 9 − 10 = −1; N = 3.83×10^{-1} = 0.383

16 TRIGONOMETRIC FUNCTIONS

A. SCALES. S scale is for sines and cosines; T scale for tangents (or cotangents); ST scale for sine or tangent of small angles.

B. READING THE SCALES. 1. Angles (θ), measured in degrees, are indicated by the numbered marks. On many slide rules, the S and T scales have two angles associated with numbered marks: θ (values of θ increase from left to right) and $(90° - \theta)$; values of $(90° - \theta)$ increase from right to left and are sometimes printed in red. **Ex:** On the S scale, mark 70 | 20 represents both $\theta = 20°$ and $(90° - \theta) = 70°$.

2. Angles not numbered on the scale are positioned by counting the number of primary marks in the space between labeled angles. **Ex:** 24° is located on the fourth primary mark between labeled angles 20° and 25°.

3. Fractions of angles are located between primary marks and may be expressed either in tenths of degrees or minutes (60 min. = 1 deg.) depending upon the make of the slide rule.

	DEGREES			MINUTES		
Number of marks	1	4	9	1	2	5
Number of spaces	2	5	10	2	3	6
Value of space	0.5	0.2	0.1	30′	20′	10′

C. TRIG. OPERATIONS

SIN θ

θ	0.57° to 5.7°	ST
sin θ	0.01 to 0.1	C

SRO: Set hln on θ on scale ST, read the value of sin θ under hln on scale C. D.P. Place 1 zero between the d.p. and the first digit. **Ex:** sin 1.62°. SRO: Set hln on 1.62° on ST, read D2-8-2. ANS: 0.0282 (**Ex:** Sin 3° 14′. SRO: Set hln on 3° 14′ on ST; read C5-6-4. ANS: 0.0564)

θ	5.7° to 90°	S
sin θ	0.1 to 1.0	C

NOTE: Use the numbers on the S scale which represent θ (usually to the right of the numbered marks). The scale reads from 5.7° on the left, to 90° on the right. The single mark between 80° and 90° represents 85°. SRO: Set hln on θ on S, read value of sin θ under hln on C. D.P.: Place to the left of the first digit. **Ex:** sin 20°. SRO: Set 20° on S at 70 | 20, read C3-4-2. ANS: 0.342 **Ex:** sin 22.3° (22°20′) SRO: Set hln on 22.3° (22°20′) on S, read C3-8-0. ANS: 0.380 **NOTE:** sin 81° is 0.988; sin 85° is 0.996; sin 89° is 0.999.

COS θ

θ	84.3° to 0°	S
cos θ	0.1 to 1.0	C

NOTE: Since sin θ = cos $(90° - \theta)$, the graduations on the S scale to the left of the numbered marks, representing $(90° - \theta)$, are used for the cosines of the angles. The cosine scale reads from 0° on the right to 84.3° on the left. The single mark between 0° and 10° represents 5° for the cosine (and also $(90° - 5°) = 85°$ for the sine). SRO: Set hln on θ on S, read value of the cos under hln on C. D.P.: Place to the left of first digit. **Ex:** cos 65°. SRO: Set hln on 65° on S at 65 | 25; read C4-2-2. ANS: 0.422 **Ex:** cos 66.4°. SRO: Set hln on 66.4° on S, read C4-0-1. ANS: 0.401 **Ex:** cos 4°. SRO: Set hln on 4° S by dividing space between 0° and 5°, read C9-9-7. ANS: 0.997

θ	89.4° to 84.3°	ST
cos θ	.01 to 0.1	C

SRO: Using the relationship sin θ = cos $(90° - \theta)$, set $(90° - \theta)$ on scale ST; read cos value on C. D.P.: Place one zero between the d.p. and the first digit. **Ex:** cos 86° SRO: Set $(90° - 86°)$ or 4° on ST, read C6-9-7. ANS: 0.0697

TAN θ

θ	0.57° to 5.7°	ST
tan θ	0.01 to 0.1	C

SRO: Set θ on ST, read value of tan θ from C scale. D.P.: Place one zero between d.p. and first digit. **Ex:** tan 3.5° SRO: Set 3.5° on ST, read C6-1-1 ANS: 0.0611

θ	5.7° to 45°	T
tan θ	0.1 to 1.0	C

NOTE: Use the graduation on the T scale to the right of the numbered marks, which represent θ. The scale reads from 5.7° on the left, to 45° on the right. SRO: Set θ on T, read the value of tan θ on scale C. D.P.: Place to the left of the first digit. **Ex:** tan 11° SRO: Set 11° on T at 79 | 11, read C1-9-4. ANS: 0.194 **Ex:** tan 11.7° (11°40′) SRO: Set 11.7° (11°40′) on T, read C2-0-7. ANS: 0.207

θ	45° to 84.3°	T
tan θ	1.0 to 10	CI

NOTE: Use the graduations on the T scale to the left of the numbered marks, which represent $(90° - \theta)$. This scale reads from 45° on the right, to 84.3° on the left. SRO: Using the relationship tan θ = 1 $\div$ tan $(90° - \theta)$, set θ on T and read the value of tan θ from the CI scale. If there is no CI scale, use tan θ = 1 $\div$ tan $(90° - \theta)$. D.P.: Place after the first digit. **Ex:** tan 55°. SRO: Set 55° on T at 55 | 35, read CI1-4-3. ANS: 1.43 **Ex:** tan 52.5°. SRO: Set 52.5° on T, read CI1-3-0-3. ANS: 1.303

θ	84.3° to 89.4°	ST
tan θ	10 to 100	CI

SRO: Using the relationship tan θ = 1 $\div$ tan $(90° - \theta)$, set $(90° - \theta)$ on ST scale and read the value of tan θ from the CI scale. D.P.: Place after the second digit. **Ex:** tan 86° SRO: Set $(90° - 86°)$ or 4° on ST; read CI1-4-3. ANS: 14.3

D. COMBINED TRIGONOMETRIC OPERATIONS. Multiplication and division involving trigonometric functions may be performed without recording the value of these functions by using the S, T, and ST scales exactly as the C scale. This is possible since the angles on the S, T, and ST scales are in line with the corresponding trigonometric functions of these angles on the C scale and the right and left indices on the trigonometric scales are in line with the indices on the C scale. **Ex:** $2.3 \times \sin 8°$ AA: $\sin 8° \cong 0.1$ $2 \times \sin 8° \cong 0.2$ SRO: Set LC1 on D2-3. Set hln on S8°. Read D3-2 under hln. ANS: 0.32 **Ex:** $0.315 \div \tan 39°$ AA: $\tan 39° = 1$; $0.3 \div \tan 39° = 0.3$ SRO: Set hln on D3-1-5. Bring T39° under hln. Read D3-8-9 under RT1. ANS: 0.389 **Ex:** $6.38 \times (\cos 58°)^2 \div 0.132$ AA: $\cos 58° = 0.5$; $6 \times (0.5)^2 \div .1 = 15$ SRO: Use A and B scales. (See Section 14: B.) Set RB1 at A6-3-8. Set hln on S58° (cosine marking). Bring B1-3-2 under hln. Read A1-3-6 on B index. ANS: 13.6

17 LOG LOG SCALES

A. DESCRIPTION. The scales labeled LL1, LL2, LL3 cover numbers >1. Numbers <1 are on reciprocal scales which may be labeled LL01, LL02, etc; or LL1−, LL2−, etc.; LL/1, LL/2, etc.; or LL0 and LL00. All scales are read with the d.p. in printed position.

B. APPLICATIONS. 1. To find natural logarithms ($\log_e$ N or ln N). SRO: **1.** Set hairline on N on appropriate LL scale. **2.** Read $\log_e$ under hairline on D [or DF/M scale]. **3.** D.P.: Located by the exponent range [or first digit position] of the LL scale used: N on LL1, $\log_e$ N on D has two decimal places (.01→0.1 or 0.0D); on LL2, one decimal place; on LL3, one digit left of the d.p. in $\log_e$ N. **Ex:** $\log_e 12.2$ Set hln on 12.2 on LL3. Read under hln D [or DF/M] 2-5-0. ANS: 2.50 **Ex:** $\log_e 0.98$ Set hln on 0.98 on LL01 [or LL1−]. Read under hln D [or DF/M] 2-0-2. ANS: Since exponent range on LL01 is −0.01 to −0.1 [or on LL1−, first digit position is −0.0D] answer is −0.0202 **NOTE:** On slide rules having only the LL0, and LL00, read $\log_e$ N on the A instead of the D scale. D.P.: N on LL0, use AA = N−1; N on LL00, left half of A scale is −0.D; right half is −D.0 **Ex:** $\log_e 0.97$ Set hln on 0.97 on LL0. Read under hln A3-0-5. AA: 0.97 − 1 = −0.03 ANS: −0.0305 **Ex:** $\log_e 0.50$ Set hln on 0.50 on LL00. Read under hln 6-9-5 on left half of A. ANS: −0.695

2. To find non-integer powers and roots. SRO: N^x = P or $\sqrt[x]{N}$ = Q. **1.** Calculate AA. **2.** Set hairline on N on appropriate LL scale, then set a C index under the hairline. **3.** Move hairline to x on C scale for powers, to x on CI scale for roots. **4.** Read under hairline on appropriate LL scale as indicated by the AA. **Ex:** 4^2 Set hln on 4 on LL3. Set LC1 under hln. Set hln on C2. Read 16 on LL3. **Ex:** $6.2^{-2.1}$ AA: $6^{-2} = 1 \div 36 = 0.03$ SRO: Set hln on 6.2 on LL3. Set LC1 under hln. Move hln to C2-1; read 0.0217 under hln on LL03, [or LL/3, or LL3−]. **Ex:** $\sqrt[4]{30}$. $\sqrt[4]{30} = 2.3$. SRO: Set hln on 3 on LL3. Set LC1 under hln. Move hln to CI4-1. Read 2.39 under hln on LL2. **NOTE:** On rules having only two reciprocal scales [LL0, LL00], use instead of scale C, scale B (or A) with the correct half to employ determined by the AA. Furthermore, negative powers of N can only be solved by first evaluating the positive reciprocal. **Ex:** $6.2^{-2.1} = 1/(6.2)^{2.1} = (0.162)^{2.1}$ AA: $(0.16)^2 = 0.026$. SRO: Set hln on 0.162 on LL0. Set central B1 under hln. Set hln on 2-1 on right half of B scale. ANS: Read 0.0217 under hln on LL00.